Diagnostics of a Subsurface Radial Outflow From a Sunspot

D. C. Braun[1]

Solar Physics Research Corporation, 4720 Calle Desecada, Tucson, AZ, 85718

Y. Fan

National Solar Observatory 950 N. Cherry Ave., Tucson, AZ 85719

C. Lindsey[1]

Solar Physics Research Corporation, 4720 Calle Desecada, Tucson, AZ, 85718

S. M. Jefferies[1]

Bartol Research Institute, University of Delaware, Newark, DE, 19716

ABSTRACT

We measure the mean frequencies of acoustic waves propagating toward and away from a sunspot employing a spot-centered Fourier-Hankel decomposition of p-mode amplitudes as measured from a set of observations made at the South Pole in 1991. We demonstrate that there is a significant frequency shift between the inward and outward traveling waves which is consistent with the Doppler effect of a radial outflow from the sunspot. For p-modes of temporal frequencies of 3 mHz it is observed that the frequency shift decreases slightly with spatial frequency, for modes with degree ℓ between 160 to 600. From the ℓ dependence of the frequency shift, we infer that the mean radial outflow within the observed annular region (which extends between 30 and 137 Mm from the spot) increases nearly linearly with depth, reaching a magnitude of about 200 m/s at a depth of 20 Mm. This outflow exhibits properties similar to flows recently reported by Lindsey, et al. (1996) using spatially sensitive local helioseismic techniques.

Subject headings: Sun: oscillations — Sun: sunspots

---

1.   Introduction

In the past several years, observational and theoretical tools have been developed in the field of local helioseismology to study the interaction of solar acoustic waves with localized structures within the solar interior. These have included the use of "ring diagrams" as probes of local convective motions (Hill 1988, Patron, et al. 1995), time-distance helioseismic measurements of sound speed and Doppler perturbations (Duvall et al. 1996, D'Silva et al 1996, Kosovichev 1996), scattering and absorption of waves by sunspots (Braun, Duvall and LaBonte 1988, Bogdan, et al. 1993, Braun 1995, Fan, Braun and Chou 1995), and acoustic power maps (Lindsey and Braun 1990, Braun, et al 1992, Toner and LaBonte 1993, Lindsey et al. 1996).

Recently, new evidence for subsurface flows associated with active regions has been presented. Duvall, et al. (1996) have used time-distance helioseismic techniques to construct spatially resolved maps of the p-modes travel times over the solar surface, using full-disk solar images obtained at the geographic South Pole in 1991. They found considerable time travel decreases in acoustic rays propagating away from active regions, which they interpreting as being due to 2000 m/s downflows within the upper 2 Mm of the convection zone. Lindsey, et al. (1996) constructed acoustic power maps of the same South Pole data, filtered in the Fourier domain to show Doppler signals caused by horizontal flows. These maps showed the presence of outflows from active regions which, for mature regions appeared to be predominately concentrated at depths greater than 8 Mm below the surface.

An important analysis procedure employed in the exploration of p-mode – sunspot interactions has been the decomposition of the solar oscillation signal, observed in an annulus surrounding a spot, into appropriate inward and outward propagating wave modes. This method has been termed "Fourier-Hankel spectral decomposition", since the annulus is usually chosen to be small enough so that the spatial (radial) form of the wave modes is described very closely with Hankel functions. The initial utility of this procedure was demonstrated by the exploration of the p-mode absorption qualities of sunspots and other solar activity by looking at the difference in amplitudes between the inward and outward wave components (Braun, Duvall, and LaBonte 1988, Bogdan et al. 1993, Braun 1995, Chen, et al. 1996). Braun, et al. (1992) and Braun (1995) have subsequently determined the scattering phase shifts and mode-mixing amplitudes due to the refractive properties of spots.

We suggest that Fourier–Hankel spectral decomposition methods are highly useful in studying horizontal flows associated with active regions. For example, a net radial inflow or outflow centered on a particular point (e.g. a sunspot) will produce equal but opposite shifts in the horizontal wavenumber (at constant temporal frequency) of p-modes traveling toward and away from the center. The signature of a radial flow would then be a shift in the position of the p-mode ridge observed in power spectra constructed alternately from the inward and outward propagating waves. This procedure has a well known analogy in the determination of internal solar rotation by the measurement of frequency differences between prograde and retrograde traveling global

p-modes. Before proceeding, we point out one important distinction between the use of the Hankel decomposition method in determining horizontal flows and its use in probing the absorbing and refracting properties of solar active regions. The amplitude and phase differences which were the primary diagnostics in the latter analyses were used to probe absorption and scattering effects which were largely confined to a area *within the inner radius of the annulus.* In contrast, the radial Doppler diagnostic we are proposing here is, to first order, only sensitive to the mean radial flow of the medium *between the inner and outer radii of the annulus.* A consequence of this is that the magnitude of the observed frequency shift will depend not only on the magnitude of the true velocity flow, but also on the choice of the annulus dimensions.

2.   Data Reduction

The data used in this analysis consists of a subset of a larger dataset obtained at the Geographic South Pole by the NSO-NASA-Bartol group (Duvall, et al. 1996) between November 1990 and January 1991. Ca II K-line images of the Sun during this time may be found in both the works cited above, as well as on the cover of the 1995 October issue of *Physics Today* (Harvey 1995). For this study, we selected a mature active region, NOAA 6431, consisting of a single, nearly circular sunspot (with umbral and penumbral radii of 4 and 9 Mm respectively) surrounded by a small region of plage. The data we utilize consists of 185 hours of observations from Jan. 1-9, 1991, with a 67% data coverage. As a control measure, a region of quiet Sun centered at the same solar latitude as the sunspot and offset 40 degrees east in longitude was also selected for an identical analysis.

The data reduction procedure is described in detail in Braun (1995). Some additional considerations regarding Fourier-Hankel spectral decomposition technique are given by Braun, Duvall, and LaBonte (1988) and Bogdan, et al. (1993). The K-line intensity values are first interpolated onto a spherical polar coordinate system (θ, ϕ) with the sunspot situated at $(\theta = 0)$. The annular region is defined by the inner and outer polar angles θ_{min} and θ_{max} respectively. For values of $\theta \ll \pi$, we may employ Hankel functions as approximations to the Legendre function decomposition. Thus, we decompose the incident and scattered waves into components of the form

$$\Psi_m(\theta, \phi, t) = e^{i(m\phi + 2\pi\nu t)}[A_m(\ell, \nu)H_m^{(1)}(\ell\theta) + B_m(\ell, \nu)H_m^{(2)}(\ell\theta)], \tag{1}$$

where m is the azimuthal order, $H_m^{(1)}$ and $H_m^{(2)}$ are Hankel functions of the first and second kind respectively, t is time, ν is the temporal frequency and ℓ is the spatial wavenumber (which may be compared with the degree of a spherical harmonic). A_m and B_m represent the complex amplitudes of incoming and outgoing waves respectively.

The values of θ_{min} and θ_{max} should, strictly speaking, be determined from the actual radial size of the velocity features one wishes to measure. Extending θ_{max}, for example, beyond the radial extent of flow adds no additional signal with the disagreeable cost of increasing the level of noise. On the other hand, practical considerations suggest the annulus should not be too small such that

the spatial resolution is insufficient to isolate p-mode ridges of adjacent radial orders. For the majority of the measurements we present here we set θ_{min} and θ_{max} to 2.5 and 11.25 degrees (30 and 137 Mm) respectively, giving a resolution in ℓ of approximately 40. The outer radius of this annulus is comparable to the horizontal extent of the outflows detected by Lindsey, et al. (1996).

The numerical transforms needed to compute the set of wave amplitudes $A_m(\ell, \nu)$ and $B_m(\ell, \nu)$ are described in Braun, Duvall, and LaBonte (1988). For each value of ℓ, we measure the mode amplitudes for individual azimuthal orders for $|m|$ ranging from 0 up to a value not to exceed $\ell\theta_{min}$, a condition required for the orthogonality of the Hankel functions (Braun 1995). For the highest wavenumbers we employ azimuthal orders up to $m = 20$.

3. Results

It is immediately apparent with only a visual examination of the power spectra that the ridges of the outgoing p-mode power are shifted to higher temporal frequencies relative to the incoming waves. The frequency shift is more readily visible when one corrects for the effects of p-mode absorption by the sunspot by individually normalizing the incoming and outgoing power. Figure 1 shows the resultant power spectra of two representative modes in the sunspot analysis. The large widths of the peaks are the result of a relatively poor wavenumber resolution. In spite of this, frequency shifts on the order of 10 μHz are clearly seen between the incoming and outgoing profiles. To measure the shift, we determine the power-weighted mean frequency for both power spectra. We define $\Delta\nu$ as the mean frequency of the outward propagating mode minus the mean frequency of the inward propagating mode. Values of $\Delta\nu$ determined from both the sunspot analysis and the quiet-Sun analysis are shown in the top and bottom panels of Figure 2 respectively. For each value of ℓ we show the frequency shift of the mode with temporal frequency closest to 3 mHz, which represent the most reliable determinations. Measurements of $\Delta\nu$ of modes with frequencies below 3 mHz or above 4 mHz showed considerable scatter in both the sunspot and quiet-Sun data. Figure 2 clearly shows positive frequency shifts for almost all modes analyzed in the sunspot data, while showing no detectable shifts in the quiet-Sun analysis. The most plausible cause of the frequency shifts is a radial outflow from the sunspot.

For values of ℓ below 240, the p-mode ridges were too close together in the power spectra to measure the frequency shift using the annulus specified above. The analysis was repeated using an annulus of twice the width as the original annulus. This enabled the detection and measurement of frequency shifts down to $\ell = 160$. A comparison of the frequency shifts measured in both the smaller and larger annuli for common modes showed that the shifts seen over the larger annulus were on average a factor of 2.8 times smaller than observed with the smaller annulus, suggesting that no flows are actually detectable beyond 137 Mm, the outer radius of the smaller annulus. The values of the shifts for ℓ between 160 and 230 determined from the larger annulus and multiplied by the factor 2.8 are shown as open circles in the top panel of Figure 2.

Doppler frequency shifts of high degree p-modes produced by horizontal flows have been previously studied (e.g. Gough and Toomre 1983, Hill 1988, Patron et al. 1995). In cylindrical coordinates the presence of a radial axisymmetric flow $U_r \hat{\mathbf{r}}$ will produce a relative frequency shift of $2k\bar{U}_r$ between the incoming and outgoing cylindrical waves, where $\bar{U}_r$ corresponds to an average of U_r over the annular region and depth weighted by the wave energy density:

$$\Delta\omega \equiv \omega^{out} - \omega^{in} = 2k\bar{U}_r = \frac{2k \int_0^\infty (\int_{r_1}^{r_2} U_r \, dr) \, K \, dz}{(r_2 - r_1) \int_0^\infty K \, dz} \tag{2}$$

where, r_1 and r_2 denote respectively the inner and outer radii of the annular region, k is the horizontal wavenumber, z denotes depth from the surface, and the kernel K is the energy density of the particular wave mode as a function of depth. In terms of our measured frequency shift $\Delta\nu$ equation (2) can be expressed as

$$\Delta\nu \equiv \nu^{out} - \nu^{in} = \frac{\ell \int_0^\infty <U_r> \, K \, dz}{\pi R_\odot \int_0^\infty K \, dz} \tag{3}$$

where $R_\odot$ is the solar radius and $<U_r>$ is the mean radial flow over the annulus.

It is straightforward to see from equation (3) that a mean flow $<U_r>$ which is, say, constant with depth will produce frequency shifts which are simply proportional to ℓ. The observed behavior of the frequency shifts at 3 mHz (Figure 2), which shows a slight decrease of $\Delta\nu$ with ℓ, implies therefore that the flows must actually *increase* with depth. Using models of $<U_r>$ with different power law increases with depth z we have computed numerically the expected frequency shifts given by equation (3) for modes of $\nu = 3$ mHz as a function of ℓ. The energy density kernels are computed from a standard solar model (Christensen-Dalsgaard, Proffitt, and Thompson 1993). We find that a mean velocity which increases as $z^{1.2}$ can explain the observed ℓ variation. The mean radial velocity is plotted in the upper panel of Figure 3, and the calculated frequency shifts are shown by the curves overlaying the data in Figure 2. The dashed line indicates a mean radial velocity which continues to increase with a power law of exponent 1.2, while the solid line represents velocity which levels off to a constant value at a depth of 20 Mm below the surface. The modes with the lowest ℓ values observed penetrate 30 Mm below the surface, so we are unable to infer velocities below this depth.

Having determined the radial outflow $<U_r>$ in the annular region as a function of depth, we can estimate the vertical flow using the requirement of the continuity equation $\nabla \cdot (\rho \mathbf{U}) = 0$, where ρ is the density. Consider the closed surface formed by an imaginary cylindrical tube extending between the surface and a depth z with its vertical axis centered on the sunspot and having a radius of r_1 equal to the inner radius of the annular region. The mass flux through the top cross section at the surface is naturally zero. The mass flux flowing into the tube through the bottom cross section is $\pi r_1^2 \rho(z) U_{vert}(z)$, where U_{vert} denotes the upflow velocity. The mass flux flowing out of the cylinder tube through the tube surface at r_1 is given by $2\pi r_1 \int_0^z \rho(z') U_r(r_1, z') dz'$, where $U_r(r_1, z)$ denotes the radial outflow velocity at r_1. Here we approximate $U_r(r_1, z)$ by the averaged $<U_r>$ in the annular region which is most likely a lower estimate judging from the radial profile

of the outflow obtained by Lindsey et al. (1996). Equating the flux through the bottom cross section to the flux through the tube surface at r_1 as required by the continuity equation, we obtain

$$U_{vert}(z) = \frac{2}{r_1\rho(z)} \int_0^z \rho < U_r > dz'. \tag{4}$$

The lower panel of Figure 3 shows the resulting $U_{vert}(z)$, which is effectively an averaged upflow velocity through the region defined by the inner annulus.

Is it possible that the frequency shifts we are observing are due to some effect which is not Doppler in nature? We note that sunspots are efficient absorbers of acoustic radiation and that a variation of the absorption with frequency will give rise to a shift in the mean frequency of the outgoing mode peak. We have measured the absorption produced by NOAA 6431, which we find to be similar to that exhibited by two sunspots analyzed in the 1988 South Pole data (see Figures 4 and 5 in Braun 1995). Of the modes represented in Figure 2, those with ℓ below 350 show an increase of absorption with frequency along the ridge, the strongest variation amounting to a 5% increase in the absorption coefficient over a 0.1 mHz frequency interval. We have estimated that this variation would produce a frequency shift, $\Delta\nu$, at $\ell = 330$ of about -2 μHz, which is of the opposite sign than that observed and has a magnitude smaller than the formal errors shown in Figure 2. The modes with $\ell > 350$ show essentially no variation in absorption along their respective ridges and are thus not affected in any detectable manner.

The outflow detected in NOAA 6431 by Hankel spectral decomposition techniques appears very similar to the outflows detected by Lindsey, et al (1996) using horizontal Doppler-sensitive acoustic power maps. The mean value of the radial velocity profile shown in Figure 5 of Lindsey et al. over our annulus size is approximately 90 m/s, which is consistent with the flow speeds inferred here. Lindsey et al. also find that for mature active regions the flows are predominately subsurface with speeds increasing with depth.

Mature sunspots are known to show surface outflows ("moats") detected by Doppler shifts and the proper motion of nearby magnetic features (a recent summary is given by Brickhouse and LaBonte 1988). These flows have typical peak velocities on the order of 500-1000 m/s and extend approximately 20 Mm beyond the penumbra. The outflows detected using local helioseismology have significantly lower velocities, and appear to persist to several times the radial extent of the moat. It is possible that these outflows may represent a subsurface extension of the moat flow.

On the other hand, the upflow required to feed the subsurface outflow appears to be at odds with the large downflows inferred by the time-distance measurements of Duvall, et al (1996). It is possible to produce wave travel time decreases along rays emanating from active regions with outflows and preliminary results using time-distance methods which provide directional discrimination appear to support this picture (Duvall, private communication). However, velocities on the order of 100 m/s appear to be an order of magnitude too small to match the observed travel times. We should not rule out the possibility of rather complex flow patterns beneath active regions. A shallow (2 Mm deep) eddy which is flowing down at the border of the magnetic flux

combined with a deeper region of upflow which drives a subsurface outflow may reconcile the helioseismic data, although an additional eddy at the extreme surface would be needed to produce the moat flow.

It is clear that the continued development and application of these and other observational techniques to the high quality data now becoming available with the GONG and SOHO projects, combined with more sophisticated theoretical modeling, will be crucial in exploring and understanding the subsurface structure and evolution of solar magnetic regions.

REFERENCES

Bogdan, T. J., Brown, T. M., Lites, B. W., and Thomas, J. H. 1993, ApJ, 406, 723

Braun, D. C. 1995, ApJ, 451, 859

Braun, D. C., Duvall, T. L. Jr., and LaBonte, B. J. 1988, ApJ, 335, 1015

Braun, D. C., Lindsey, C., Fan, Y. and Jefferies, S. M. 1992, ApJ, 392, 739

Brickhouse, N. S. and LaBonte, B. J. 1988, Solar Phys., 115, 43

Chen, K.-R., Chou, D.-Y., and the TON team, 1996, ApJ, in press

Christensen-Dalsgaard, J., Proffitt, C. R., and Thompson, M. J., 1993, ApJ, 403, L75

D'Silva, S., Duvall, T. L. Jr., Jefferies, S. M., Harvey, J. W. 1996, ApJ submitted

Duvall, T. L. Jr., D'Silva, S., Jefferies, S. M., Harvey, J. W., and Schou, J. 1996, Nature, 379, 235

Fan, Y., Braun, D. C. and Chou, D. 1995, ApJ, 451, 877

Gough, D. O. and Toomre, J. 1983, Solar Phys., 82, 401

Harvey, J. 1995, Physics Today, 48 (No. 10), 32

Hill, F. 1988, ApJ, 333, 996

Kosovichev, A. G. 1996, ApJ, in press

Lindsey, C. and Braun, D. 1990, Solar Phys., 126, 101

Lindsey, C., Braun, D. C., Jefferies, S. M., Woodard, M. J., Fan, Y., Gu, Y., and Redfield, S. 1996, ApJ, submitted

Patron, J., Hill, F., Rhodes, E. J., Korzennik, S. G. and Cacciani, A. 1995, ApJ, 455, 746

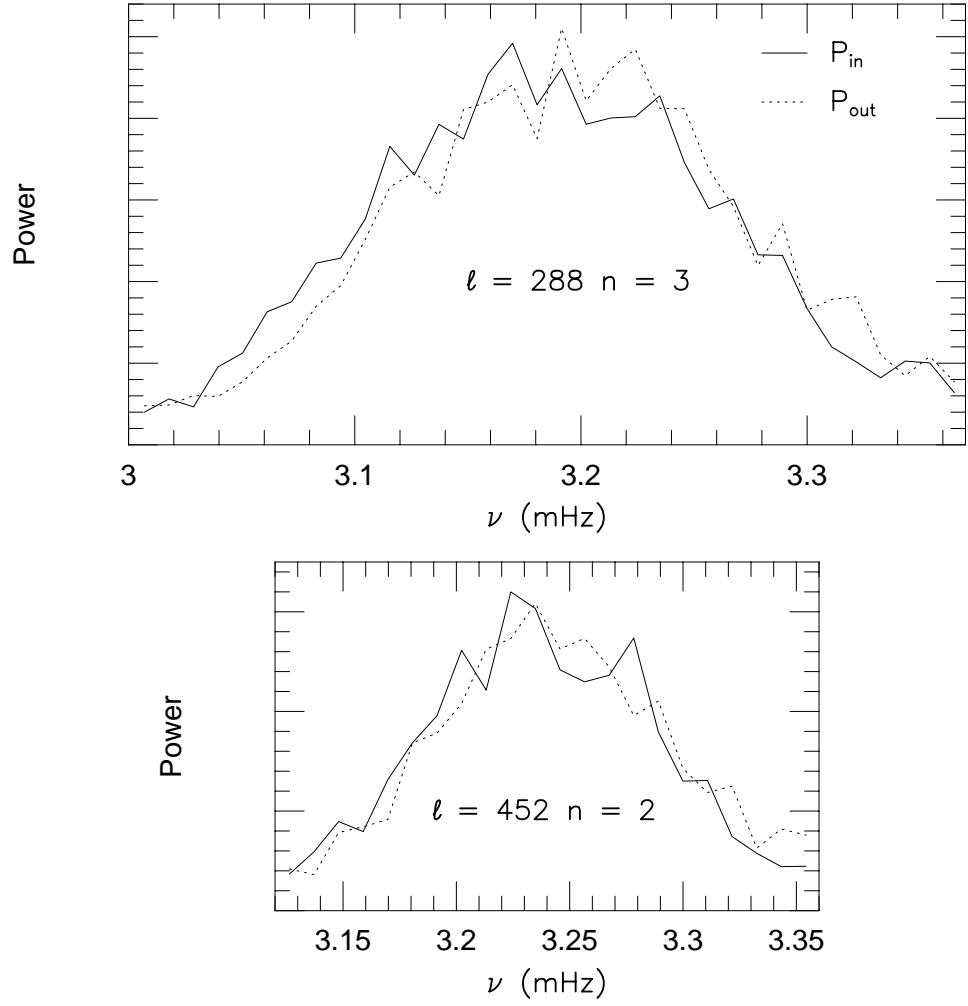Toner, C. G., and LaBonte, B. J. 1993, ApJ, 415, 847

Fig. 1.— Power spectra corresponding to inward and outward propagating p-modes for two representative modes. A relative shift in the peak of the outward modes towards higher frequency is apparent in both cases.
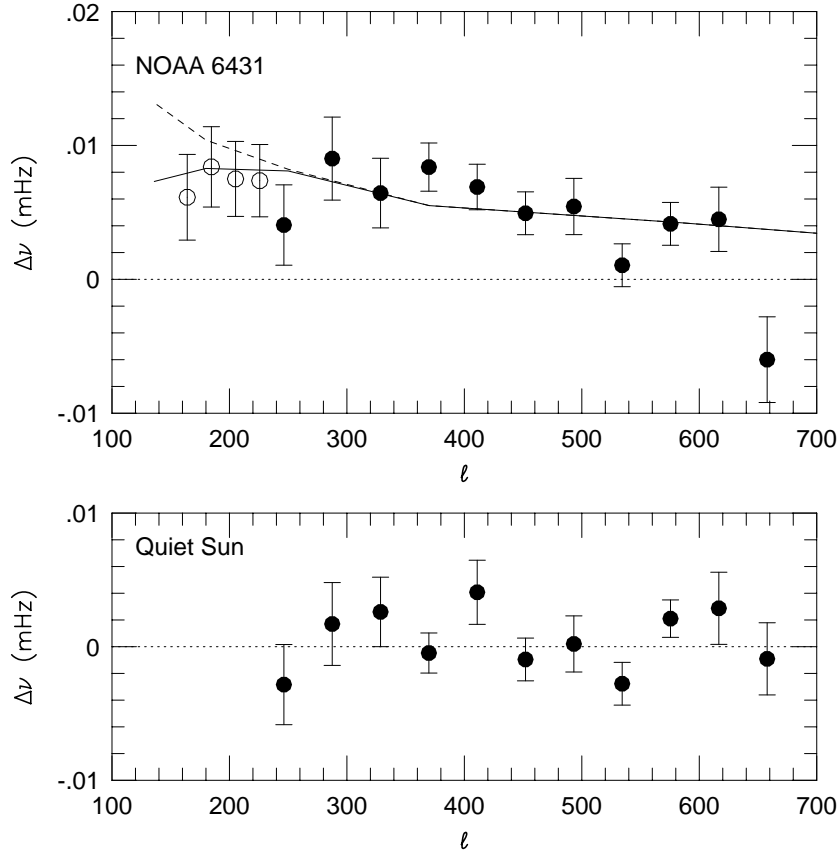
Fig. 2.— Measurements of the frequency shift plotted as a function of degree ℓ for the sunspot (top panel) and quiet-Sun (bottom panel) analysis. The filled circles (with formal 1 σ error bars) represent measurements made over the small annulus (2.5 to 11.25 degrees from the center) while the open circles in the upper panel represent measurements made with a larger annulus (2.5 - 20 degrees), scaled by a factor of 2.8. The curves show calculated frequency shifts for two models of the mean radial outflow.
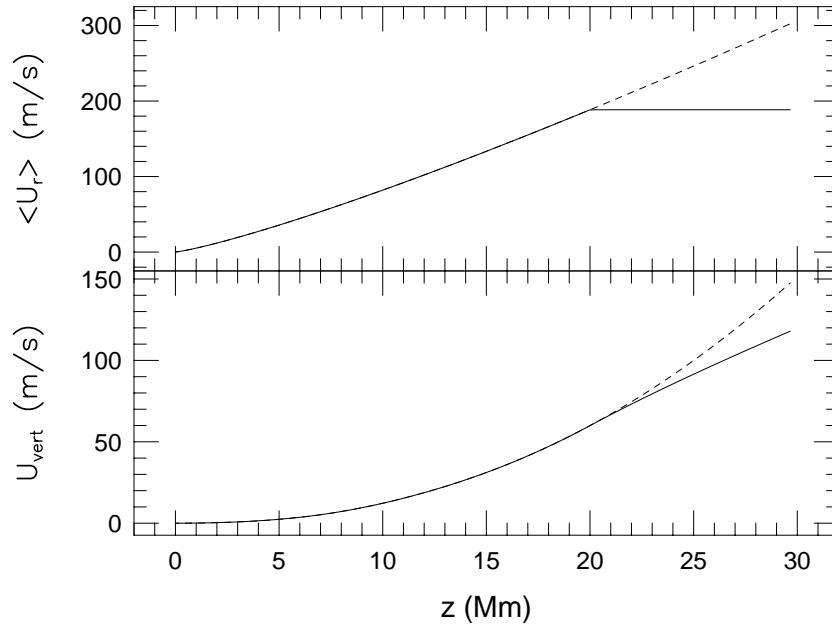
Fig. 3.— The top panel shows the inferred mean radial outflows plotted against depth. The solid and dashed curves show the outflows used to calculate the frequency shifts shown in corresponding line types in Figure 2. The bottom panel shows the upward vertical velocity inferred by flux conservation.